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Informal Report

STUDIES ON THE RCMS RF SYSTEM

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STUDIES ON THE RCMS RF SYSTEM

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Abstract. This note addresses the various options for the Rapid Cycling Medical Synchrotron (RCMS) RF. The study was divided into three cases, namely non-tuning, tuning and filter. Each case also includes a few options. The primary study was focused on the non-tuning options. However, it was found that it requires too much driver power to cover the wide band and thus causes the cost being too high to be competitive. The proposal of RCMS is not yet clear if it can be approved or not. The results of this study might be useful to other similar machines.

INTRODUCTION

A new scheme of RCMS (Rapid Cycling Medical Synchrotron) was proposed in BNL.[1,2] In the preliminary design,[3,4] the circumference of the synchrotron is 24.2m, the injected beam energy is 7MeV, and the ejected energy is 70 to 250MeV. Consequently, the synchrotron frequency should ramp from 1.5 to 7.6 MHz. The acceleration voltage required is about 3kV, a quite modest value.

Since the intended application is for use in hospitals, other essential requirements are keeping the cost competitive and the ease of handling and maintenance-free operation.

Ferrite has been widely applied for tunable cavities. Since ferrites have a natural inertia of their permeability, a repetition rate of 30 Hz seems too high, so much effort was focused on the non-tuning scheme at first.

Unfortunately, though the gap voltage was quite modest, the non-tuning scheme still requires too much power, because the ramped frequency band is too wide, say 5 to 1. This suggested a comprehensive investigation of various options.

Many possible schemes have been investigated. This note addresses them in order, and summarizes their advantages and disadvantages.

SPECIFICATIONS

In order to optimize the RCMS design, the parameters had a few iterations. Finally, the major parameters related to RF are listed below:

Number of cavities	1 to 3
Maximum cavity voltage	6.5 kV
Inject beam energy	7 MeV
Ejected beam energy	70 - 250 MeV
Circumference	28.6 m
Revolution frequency (inj)	1.274 MHz
Revolution frequency (top)	6.44 MHz
Repetition frequency	30 Hz

The beam current is quite low and thus the beam loading is negligible in the cavity. The revolution period has to be consistent with the field of the dipole magnets, which is sinusoidal. Consequently, the gap voltage and frequency must be ramped such as to match the ramp curve of beam energy. The ramp curve is shown in Fig.1.

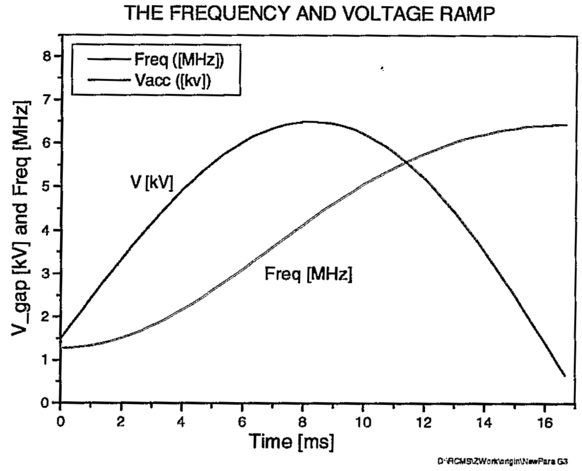

Fig.1 The ramp curves of the gap voltage and the frequency

Regarding the RF design, we are more concerned with the voltage response with the frequency as shown in Fig.2.

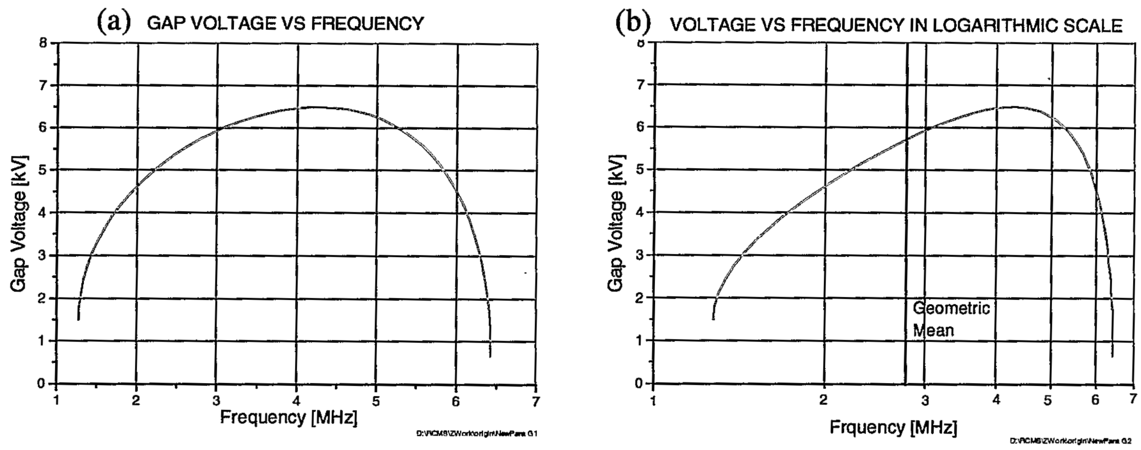

Fig.2 The cavity voltage requirement vs frequency

The frequency axis of Fig. 2(a) is on a linear scale. In a wide band circuit, the center frequency does not refer to the arithmetical mean, but geometrical mean. Therefore, a logarithmic scale is appropriate as shown in Fig. 2(b). Obviously, the required response is quite asymmetric.

GENERAL CONSIDERATION

The required voltage is only a few kilovolts, which is by no means high if it is fed to a simple resistant load. The load in question is a cavity filled with ferrite rings. It can be classified into two categories: resonant and non-resonant. For resonant cases, the load impedance is high and thus the needed drive power is low, but it can hard to cover a wide band without appropriate tuning. For non-resonant cases, the load is inductive. Since the reactive power is much higher than the loss, one would expect the required power source is much higher.

To alleviate the requirement on the power source, one way is to increase the inductance. This in turn requires increasing the number of ferrite rings or the number of cavities. However, doing so is limited by the space.

There are many possible options for both tunable and non-tuning schemes. A wide band filter structure using circuit synthesize technology is also possible. The following list summarizes these options.

Non-tuning schemes:
- Direct coupling -- tube amplifier
- Capacitance coupling -- multi-band staggering
- Inductance coupling -- tape coupling
- Transformer

Tunable schemes:
- Mechanical tuning -- rotational capacitor
- Electrical tuning -- biasing ferrite tuning

Filter -- wide band impedance matching

This note will discuss these options. To begin we discuss some common concerns.

FERRITE AND CAVITY DESIGN

The cavity was modeled on the SNS cavity for simplicity. The size sets a limit to the outer diameter of ferrite rings no more than 50cm, and the total length for ferrite has to be limited within 90cm.

Regarding ferromagnetic material, we would choose Phillips' ferrites 4M2 and 4L2 only. The AGS has had much experience with it and nothing else was found better. The outer and inner diameters are 50 and 25 cm respectively. The thickness is 2.7 cm. The parameters of the ferrite are shown below[1]:

Table 1

Material		4M2	4L2
Permeability	μ_i	140±30	250±50
	μ_{rem}	130	200
$\mu_r Q$ @2.5MHz,5mT 10mT 20mT		$20\cdot10^3$ $20\cdot10^3$ $15\cdot10^3$	$25\cdot10^3$ $20\cdot10^3$ $9\cdot10^3$

[1] The data are quoted from data sheets of Phillips[5]. There exists other data source from experiments showing differences. However, the permeability dispersion of the products may be as much as ±20% or more, so those data can only be regarded as an estimation. Trimming is always necessary for an individual device.

$\mu_r Q$ @ 5MHz, 5mT 10mT 20mT 30mT		$15\cdot10^3$ $15\cdot10^3$ $10\cdot10^3$ $7\cdot10^3$	$15\cdot10^3$ $11\cdot10^3$ $5\cdot10^3$ $2\cdot10^3$
$\mu_r Q$ @ 10MHz,5mT 10mT		$12\cdot10^3$ $10\cdot10^3$	- -

To begin with, we have to figure out the necessary minimum number of rings.

The gap voltage is simply the time differential of the total linked flux, i.e. (ignore the sign hereafter),

$$V = \frac{\partial \Psi}{\partial t} = \omega\Psi = \omega \int B dA. \tag{1}$$

The flux in the air is negligible in comparison with that in the ferrite. At low frequency the current along the cavity axis can be regarded as invariant. Then, the above integral is

$$\int B dA = l \int_{r_1}^{r_2} \frac{\mu I}{2\pi r} dr = l \frac{\mu I}{2\pi} \ln \frac{r_2}{r_1}, \tag{2}$$

where r_1 and r_2 are inner and outer radius of the ferrite ring respectively, $l = Nd$ is the total length of the ferrite stack, and N is the number of rings. It should be noted that the above formula has implicitly assumed that the permeability is constant. This would not be true in the case that a bias field is applied on the ferrite. In this note we discuss only the non-tuning case without bias. The tunable case is detailed in a separate note [6].

It is desirable to restrict the maximum flux density within the limit,

$$B_{max} = \mu I / 2\pi r_1, \tag{3}$$

equation (2) can be rewritten as

$$\int B dA = B_{max} l r_1 \ln \frac{r_2}{r_1}$$

$$V = \omega B_{max} l r_1 \ln \frac{r_2}{r_1} \tag{4}$$

Substituting $l = Nd$, the minimum number of ferrite rings would be

$$N \geq N_{min} = \frac{V}{\omega B_{max}\, d\, r_1 \ln(r_2 / r_1)} \tag{5}$$

Assuming B_{max} = *100 Gs = 10 mT*, and substituting the geometrical parameters, r_2/r_1 = 50/25, we get

$$N_{min} = 6.8\times10^3 \frac{V[V]}{f[Hz]} = 6.8 \frac{V[kV]}{f[MHz]} \tag{6}$$

Taking into account the specified voltage shown in Fig.2, the minimum number required for various frequencies is shown in Fig.3

We see that 16 rings are enough with the maximum flux at 1.79MHz.

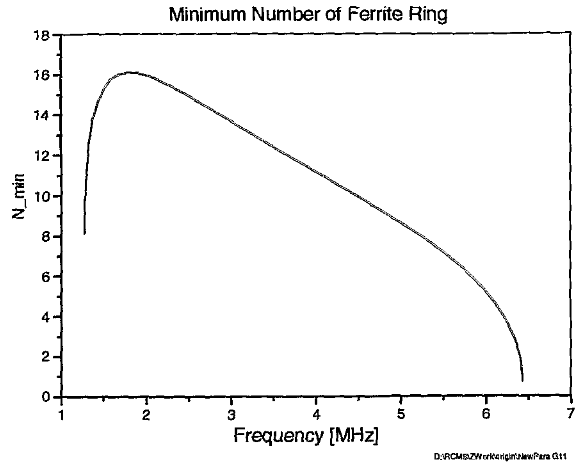

Fig.3 Minimum number of ferrite rings needed

If one cavity can contain 16 rings, then it can sustain the total required voltage. It is independent of the permeability, so the calculated ring number is applicable for both 4M2 and 4L2.

Once the number of the rings is decided, one can check the power loss in it.

The total loss of the cavity consists of loss in ferrite and in copper, but the latter is negligible in comparison with the former. Since the loss in the ferrite is non-uniform in both space and time, there are a few concerns as follows:

(a) The spatial maximum loss;

(b) The spatial average loss, which determines the real power delivered from the power source;

(c) The time average total loss, which determines the total heat to be removed by the coolant;

(d) The time average maximum loss, which checks if any place is being overheated.

The power loss density in the ferrite is

$$P_d = \frac{\pi}{\mu_0} \cdot \frac{B^2 f}{(\mu_r Q)} = \frac{2.5 \cdot 10^6 B^2 f}{(\mu_r Q)} \qquad [\frac{W}{m^3}] \tag{7}$$

$\mu_r Q$ is a complex function of flux density and frequency as seen by Table 1. Note that here μ_r is a relative value and $\mu_r Q$ is dimensionless. For simplicity, in numerical calculations we assume $\mu_r Q = 10^4$, which is equal to or less than that in the Table 1 when $B \leq 10mT$, except an uncertainty of 4L2 with $f > 5\ MHz$.

From (4) and (7) the maximum flux and power loss density are

$$B_{\max} = \frac{V}{2\pi f\, N\, d\, r_1 \ln(r_2 / r_1)} = 68 \times \frac{V}{fN} \ , \tag{8}$$

$$P_{d\max} = \frac{1}{4\pi\mu_0 [d\, r_1 \ln(r_2 / r_1)]^2} \cdot \frac{V^2}{fN^2(\mu Q)} = 1.16 \cdot 10^{10} \frac{V^2}{fN^2(\mu Q)} \quad [\frac{W}{m^3}] \tag{9}$$

The total power loss is simply a volume integral, keeping in mind that $P_d = P_{dmax} \cdot (r_1/r)^2$.

$$P_{tot} = P_{d\max} \cdot 2\pi N d\, r_1^2 \ln(r_2 / r_1) = \frac{1}{2\mu_0 d \ln(r_2 / r_1)} \frac{V^2}{(\mu Q) N f}$$
$$= 2.126 \times 10^3 \frac{V^2}{Nf} \quad [W] \tag{10}$$

The average power loss density is

$$P_{av} = \frac{P_{tot}}{\pi (r_2^2 - r_1^2) N d} = k P_{d\max} \tag{11}$$

where $$k = \frac{2\ln(r_2 / r_1)}{(r_2 / r_1)^2 - 1} = 0.4621$$

Note that both frequency and voltage are functions of time. The maximum and average power density and the total power loss versus time are shown in Fig.4.

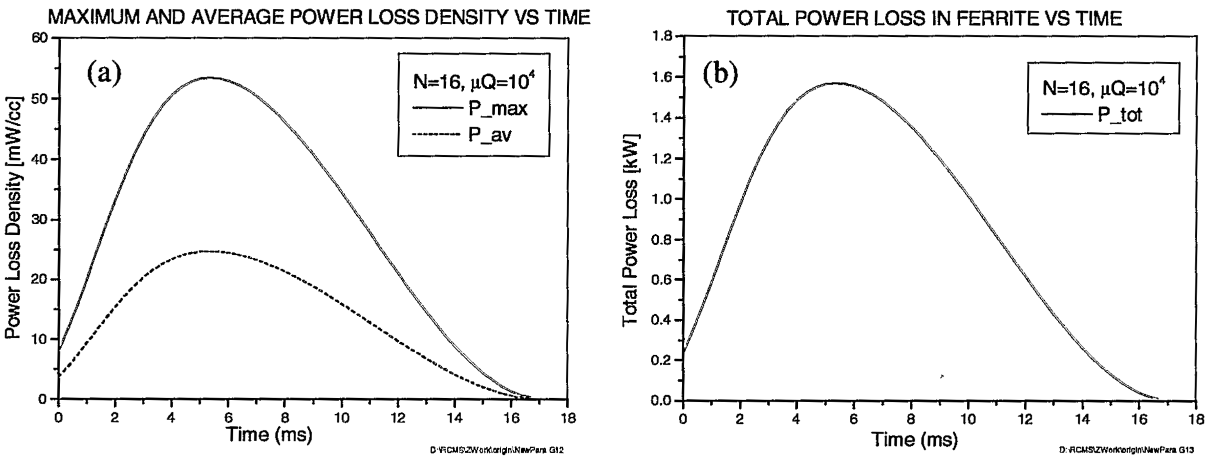

Fig.4 The power loss vs time

It is clear that the power loss is quite modest. The maximum density is less than 60 mW/cm^3, while the maximum total loss is about 1.6 kW around 2.9 MHz. Keep in mind that the data are based on the assumption that $N = 16$ and $\mu Q = 10^4$, namely we chose the minimum number of rings and a rather conservative permeability. If the cavity consists of more rings, the loss will be reduced further.

From the time dependence curve, one can figure out the time average power. In Fig.4, the average power is 880 W, while the maximum power density is 30 mW/cm^3 in time average. As heat is a concern because the duty cycle is 50 %, the total average power is then only 440 W and the hottest spot is 15mW/cm^3.

Therefore, we can conclude that the power dissipation in the ferrite is quite modest, even in the case of only one cavity with 16 rings.

Now, we can calculate the inductance. Since dissipation is worry free, the inductance is more important in our case. It depends on material and may require more rings.

The inductance is easily deduced from (3) and (4):

$$L = \frac{V}{\omega I} = l\frac{\mu}{2\pi}\ln\frac{r_2}{r_1} = N\,d\mu_0\frac{\mu_r}{2\pi}\ln\frac{r_2}{r_1} \qquad (12)$$
$$= 3.74\times10^{-9}N\mu_r$$

Obviously L is proportional to the number of rings and the permeability. However, one should keep in mind that μ_r is assumed constant here. In the biased case μ_r and L will decrease. Moreover, μ_r is a function of frequency, and so is L. Table 2 shows the inductance with different numbers of rings.

Table 2 The inductance with zero bias (μH)

Material		4M2	4L2
Permeability μ_r		100 - 140	200 - 250
Number of Rings N	14	5.2 - 7.3	10.5 - 13.1
	16	6.0 - 8.4	12.0 - 15.0
	18	6.7 - 9.4	13.5 - 16.8
	20	7.5 - 10.5	15.0 - 18.7
	22	8.2 - 11.5	16.5 - 20.6
	24	9.0 - 12.6	18.0 - 22.5

POWER REQUIREMENT

The power source must meet the requirement of both frequency and voltage ramp as shown in Fig.1. The power loss shown in Fig.4 is under these ramp conditions. However, it is roughly equal to the output power of the source only if the cavity is tuned synchronously with the frequency ramp and the source matches the cavity load for all frequencies. Otherwise, the power requirement is always larger.

Generally, a simple cavity can be represented by a L-C-R resonator. Fig.5 shows the equivalent circuit and its power supply. V_g is the gap voltage. R_s is the internal impedance of the source and I_s the source current.

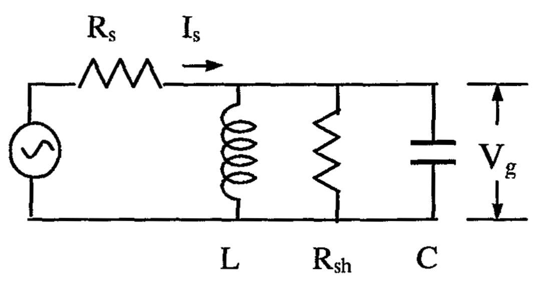

Fig.5 The equivalent circuit of a simple cavity

Obviously, the power supply must have the capability to drive a voltage of V_g and a current of I_s simultaneously. The nominal power (RMS) of the power supply should be no less than:

$$P_{nom} = \tfrac{1}{2}V_{g\,\max}\times I_{s\,\max} \quad [VA] \qquad (13)$$

It is worth noting that both V_g and I_s should be their maximum in the whole band, although they may not occur in the same frequency.

If one chooses N cavities with equal parameters instead of one cavity, then the voltage of each gap will be N times less. So is the drive current I_s. Then the drive power for each gap will be:

$$P_{nom.ea} = \frac{1}{2}\frac{V_{g\max} I_{s\max}}{N^2}, \tag{14}$$

where V_{gmax} is the total voltage in a cycle. The total power required is:

$$P_{nom.tot} = N \cdot P_{nom.ea} = \frac{1}{2N} V_{g\max} I_{s\max} \tag{15}$$

Evidently, the more cavities, the less power required. Therefore, it's better to choose as many cavities/gaps as possible as long as the space and other factors permit.

The total voltage per cycle has been determined by beam dynamics as mentioned at beginning and can't be changed. So, the power is determined by the current. On resonance, the drive current is low. But, in the non-tuning case the current is much larger. Since the non-tuning scheme is of special interest, we'll calculate the current.

Obviously, the larger the inductance, the smaller the drive current. So we should design a cavity with as large an inductance as possible. From Table 2 the material 4L2 is preferable, though it is not recommended to operate higher than 5MHz.

Now, let's estimate quantitatively the required power. Assume that there are two cavities (two gaps) in the ring and the inductance is 19 μH per gap. The capacitance is 120 pF. Conforming to the voltage requirement of Fig.2, the drive current as a function of frequency is shown in Fig.6 and the related reactive power is shown in Fig.7.

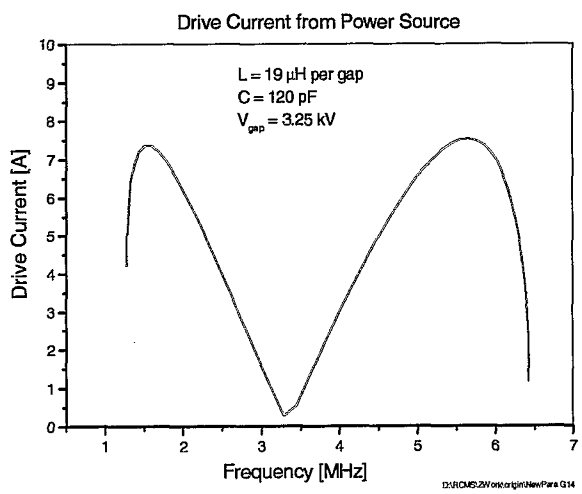

Fig.6 The drive current vs frequency

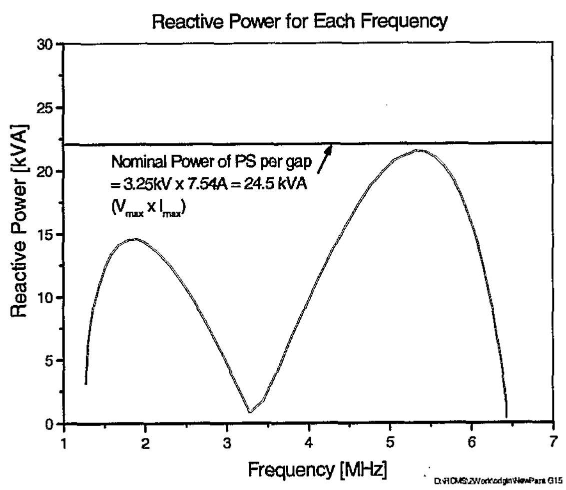

Fig.7 The reactive power vs frequency

Note that the maximum current is about 7.5 Ampere at 5.7MHz. The maximum reactive power is 21.5 kVA at 5.3 MHz. The power supply should be capable of reaching a maximum current of 7.5 A and a maximum voltage of 3.25 kV simultaneously. Then the nominal peak power is 24.5 kVA. Since an amplifier normally specifies its output power in RMS, it requires an amplifier with a RMS power of 12.25 kW. The two cavities need a total power of 24.5 kW, providing the impedance match between supply and cavities is optimized.

AMPLIFIER ISSUE

From the foregoing, the maximum real power loss is 1.6 kW in a non-tuning cavity case, while the reactive power is 24.5 kW, much higher than the former. If the coupling between amplifier and cavity is not properly matched, the specified power of the amplifier should be even higher. The questions arise:

(1) How to specify the power of an amplifier? Specifically, is a 5kW amplifier able to drive a load with reactive power of 5kVA?

(2) As the power requirement of an amplifier depends on the VSWR of the load, what is the VSWR in question?

In the frequency of a few MHz range, the most popular solid state amplifiers on the market are usually classified into class A and class AB. (Amplifiers with tubes will be discussed later.)

Class A amplifiers are linear amplifiers that are "load tolerant", that is insensitive to the VSWR of the load, but expensive. On the contrary, class AB amplifiers are cheap but very sensitive to the load.

Class-A median power amplifiers can deliver full power with any load. High power amplifiers can deliver full power with a load VSWR up to 6:1 but will reduce beyond that. Fig. 8 (top curve, 1kW or higher) illustrates the general limits.[1] For class AB amplifiers (left curve), the power has to foldback rapidly. It can also be estimated by the formula[2]

$$Deliver\ Power = \frac{Specified\ Full\ Power}{VSWR} \qquad (16)$$

This means a nominal 5kW amplifier of class-AB driving a load with VSWR of 2:1 can deliver only 2.5kW and with VSWR of 4:1, only a quarter, or 1.25kW.

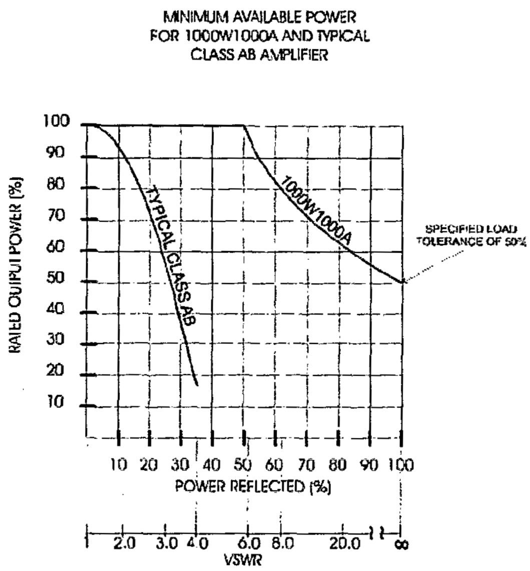

Fig. 8 Available power vs VSWR

The real capability may depend on the design, so one has to consult the manufacturer in order to fit a particular system.

Generally speaking, class A amplifiers are suitable for unmatched loads, while class AB amplifiers are suitable for matched or near matched loads.

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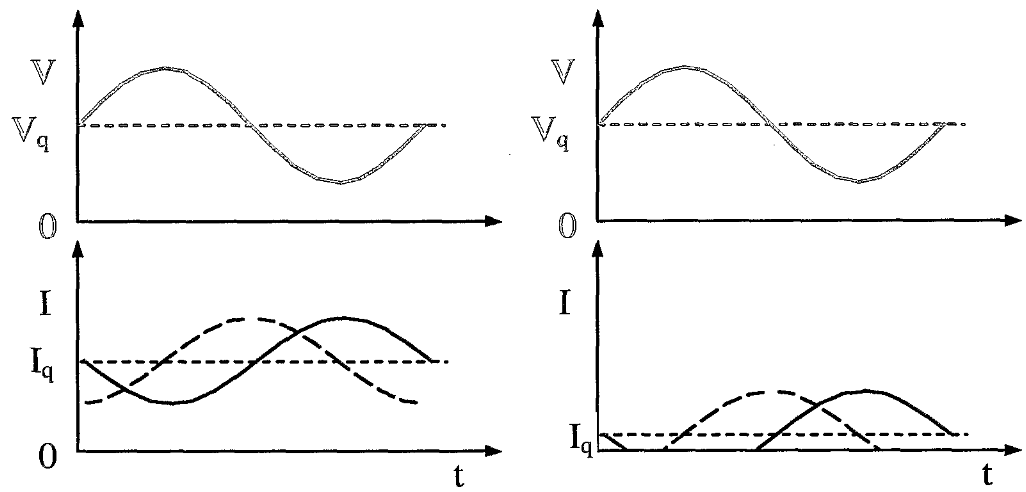
[1] Quoted from the Application Notes of Amplifier Research Inc.

[2] Bent Meier, Cubic Communications Inc. private communication.

The main reason for this difference is because they have different capabilities for heat dissipation. The heat or power dissipation is the integral of the product of voltage and current in the collector of the transistor (or the plate of the tetrode in the case of tube amplifiers).

$$P = \frac{1}{T}\int VIdt \tag{17}$$

This can be qualitatively illustrated by Fig.9. Class A amplifiers are so designed that the quiescent point lies within the linear region (see Fig.9a). Without an input signal, the power dissipated in the collector is simply $P = V_q \times I_q$, which equals the DC power provided from the supply. When an RF signal is applied, the amplifier delivers a part of the power to the load, and thus reduces the dissipation power on the collector. For a reactive load, the collector will absorb the reflected power, but the total amount of dissipation is not larger than that of the quiescent state and is thus tolerable.



(a) Class A (b) Class AB

Fig. 9 The voltage and current on the collector. Dot straight line is quiescent state. Dashed current curves correspond to a reactive load.

On the other hand, in a class AB amplifier (see Fig. 9b), the quiescent current is much smaller. V_q and I_q are close to the cut-off point. When the amplifier delivers full RF power to the load, the total dissipation may be a few times larger than that of quiescent state. With a matched load, the current is just in the opposite phase with the voltage. As shown in the figure, the maximum current occurs simultaneously with the minimum voltage. This minimizes the dissipation, and the heating is manageable.

When reflection power occurs, as in the case of an inductive load, the current phase may shift up to 90 degrees as the dashed line shown in Fig.9b. According to the integral (17), the dissipation will be larger than the designed value, even if the amplitudes of the voltage and current remain the same. (The real amplitude of the RF voltage may change from case to case, which may cause an extra contribution).

Therefore, a class AB amplifier with nominal power of 5kW can deliver 707 V and 14 A to a matched load, but can not deliver the same amount, or 5kVA, to a reactive load. This explains qualitatively the reduction in Fig.8.

Obviously, a class A linear amplifier is superior to a class AB amplifier for cases of reactive load. A load off-resonance is exactly a reactor. The VSWR can be very large. Therefore, class A is preferable.

However, because a class A amplifier is designed for a higher dissipation capability, it requires more active devices and is thus generally larger and more expensive than a class AB amplifier. Table 3 lists some data for comparison.

Table 3

Company	Type	Class	Power	Freq.[MHz]	Solid/Tube	Price
Cubic	PA5K-30	AB	5kW	1.6-30	Solid	~$80k
Communications	PA10-30	AB	10kW	1.6-30	Solid	~$160k
AR (Amplifier	5000A250	A	5kW	0.1-250	Solid	$225k
Research)	10000A250	A	10kW	0.1-250	Solid	$445k
	10000L	A	10kW	0.01-100	Tetrode	$280k

As can be seen, class A amplifiers are priced almost 3 times higher than its counterpart of class AB. How to trade off the performance and the price is of serious concern. Note also the lowest frequency of Cubic Communications Inc.'s amplifiers is 1.6 MHz, which does not meet our need in the low frequency end, say 1.27 MHz.

TUBE OPTION --- DIRECT COUPLING

Usually, a solid state source is preferable because it is compact, simple and needs little maintenance. However, from the discussion in preceding section, one can conclude that the amplifier must be class A, which is expensive. Then one has to ask if the option of a tube amplifier is worthy for comparison.

Technically, it likely has least problem, because BNL did have much experience with it. Both voltage and current have plenty of margin, so that one cavity would be adequate. In addition, because a tube amplifier applies high voltage, its output is more than the specified 6.5kV and it can be connected directly to the cavity gap, which significantly simplifies the structure. Moreover, it is cheaper than the solid amplifier as shown in Table 3 and Table 6 (see later).

However, the disadvantages of the tube option are also obvious. The support system is rather complex. For a tetrode, besides the plate voltage supply, there are screen, grid bias and heater power supplies too. It also requires a sizable driver amplifier and cooling system. More important is the need of maintenance, including the fact that the tube lifetime is limited and requires timely replacement. This is unlikely to be acceptable in a hospital.

CAPACITOR COUPLING AND MULTI-BAND STAGGERING

Unlike tube amplifiers, solid state amplifiers are usually designed to feed a 50 ohm load. Some of them are listed in Table 3. Consequently, the output voltage is rather low. A 5kW amplifier can deliver only 0.7kV. In order to get the specified gap voltage, we need to enhance the voltage somehow. Transformers are common devices to enhance voltage. Other possible options are capacitor coupling and inductance coupling. We discuss capacitor coupling here at first.

Fig. 10 shows a schematic circuit of capacitor coupling. The amplifier is represented by a voltage source V_s and the internal resistance R_s. A 5kW amplifier can deliver 0.7kV when the load is matched, so the maximum output voltage is V_{0max} = 0.7kV. But, the equivalent voltage V_s is not clear. With a matched load the load voltage is 0.7kV, and the source voltage would be doubled, say $V_s = 2V_{0max}$ = 1.4kV. With an unmatched load the voltage V_0 at the port is no more than 0.7kV, so V_s should be less than $2V_{0max}$ but larger than V_{0max}. The exact value may depend on the structure of the amplifier.

The load in question is a cavity with ferrites and represented by L and R_L. The latter accounts for the loss. The capacitors and the inductance form a certain resonance and thus the load voltage V_L will be larger than the source voltage V_s. Changing the ratio of the two capacitors C_2/C_1 = **n** will change the voltage enhancement factor, or voltage gain $G = V_L/V_s$, but not linearly.

Considering our particular case, assume $L = 20mH$, $R_L = 20\ kohm$, $R_s = 50\ ohm$, and $C_0 = 180\ pF$ = constant, where

$$C_0 = \frac{C_1 C_2}{C_1 + C_2}$$

For different ratio **n**, the voltage gain is plotted in Fig.11.

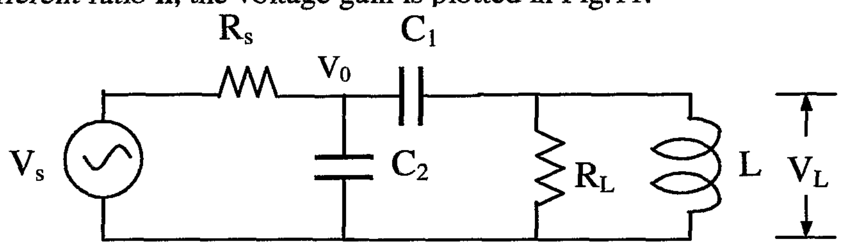


Fig.10 Schematic circuit of capacitor coupling

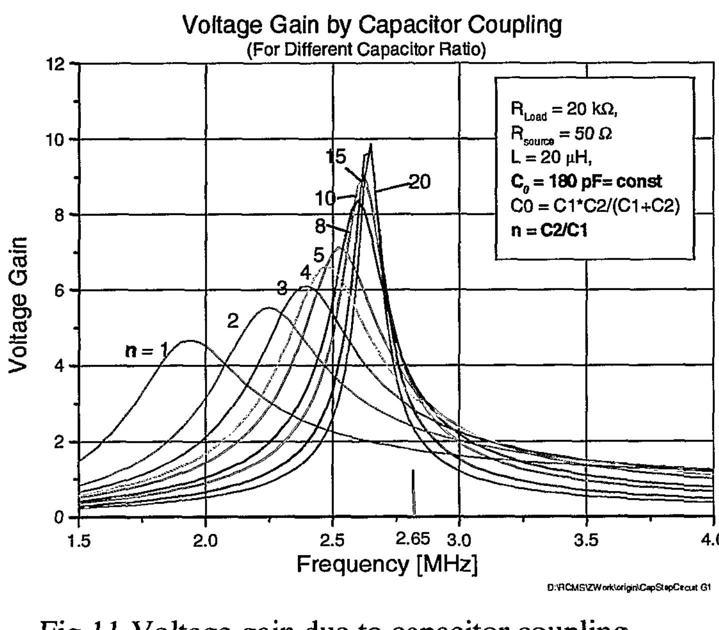


Fig.11 Voltage gain due to capacitor coupling

Evidently, the voltage gain increases with **n,** but not linearly. The maximum voltage gain is 7.1 with **n** = 5, and 6.6 with **n** = 4. However, the bandwidth does not meet the requirement.

In order to cover the full band specified in Fig.2, one may make use of the staggering method, which employs a few cavities with each tuned to a different frequency.

Table 4 gives a data set. In this example we divide the full band into 6 sub-bands (column 2) with each tuned to its center frequency (column 3). Next we allot different numbers of ferrite rings (column 4) to each band, such that each cavity has an appropriate inductance (column5). The corresponding capacitances can be calculated and are listed in the last three columns. The Q values and R_L are estimated according to their frequencies. Fig.12 plots the individual response and the summed gap voltage.

Table 4

No.	f_1 - f_2 MHz	f_0 MHz	#Ring	L* μH	Q	R_L kohm	C_0 pF	C_2 pF	C_1 pF
1	1.27-1.67	1.45	18	27	120	30	446	2231	558
2	1.67-2.18	1.9	13	19.5	110	26	360	1799	450
3	2.18-2.86	2.5	10	15	100	24	270	1351	338
4	2.86-3.75	3.3	8	12	85	21	194	969	242
5	3.75-4.91	4.3	6	9	70	17	152	761	190
6	4.91-6.44	5.6	5	7.5	55	14	108	538	135

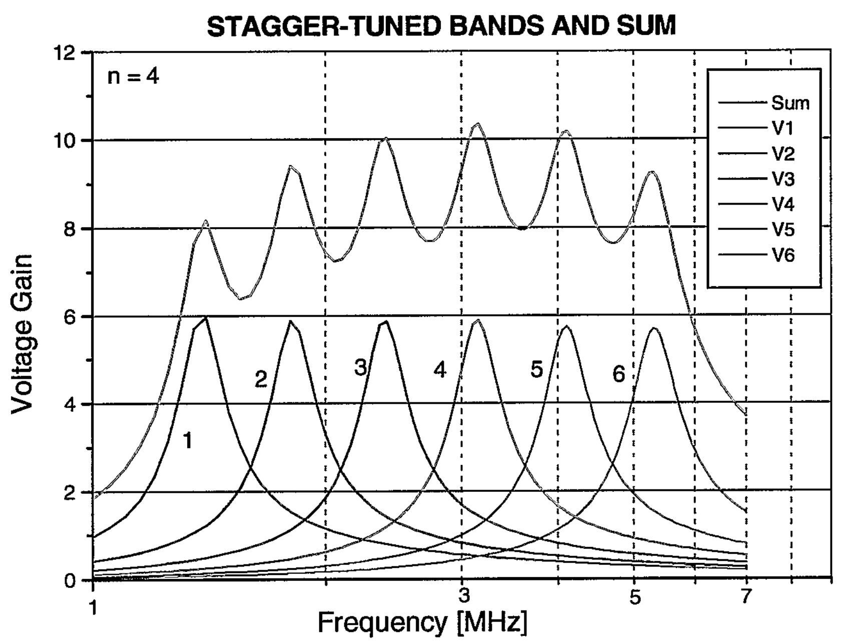


Fig.12 The staggering tuning scheme with capacitor coupling

It can be seen that the full band is covered with 6 staggering sub-bands. According to the number of rings in Table 4, one can make proper combination that one cavity contains two gaps, namely forming two sub-bands. Thus three cavities would be adequate. It is also possible to employ two cavities to cover the full band after optimization.

INDUCTANCE COUPLING AND TRANSFORMER

In principle, inductance coupling has similar performance as capacitor coupling, except that the structure is quite different. The power feed loop involves only a part of the ferrite rings. This arrangement, as well as a capacitor coupling, will not only enhance the voltage, but also improve the impedance match.

Fig. 13 shows a schematic wiring diagram. It involves two amplifiers in a push-pull driving mode. The capacitance C consists of the gap capacitance with the addition of an external capacitor in order to tune to a proper frequency. We divide the ferrite rings into 4 groups with different flux Ψ. They are also coupled by means of a "figure 8" loop as shown at the top. The arrows denote the current directions.

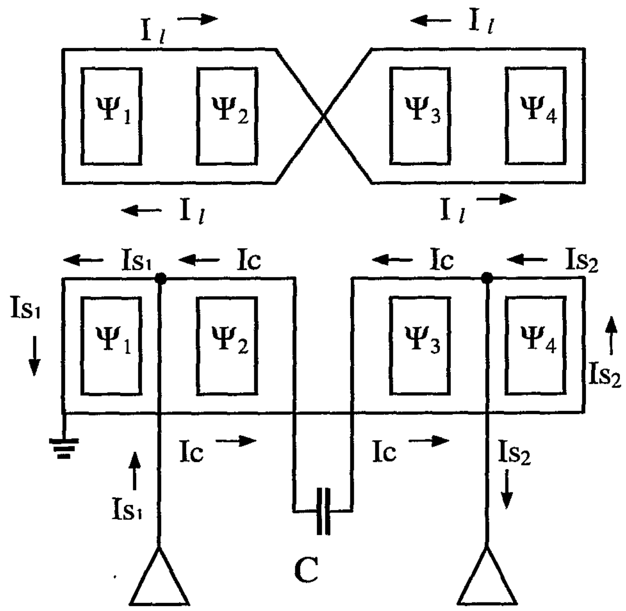


Fig.13. The wiring diagram for inductance coupling

As can be seen from Fig.13, the drive currents I_{s1} and I_{s2} of the amplifiers link only a part of the fluxes, Ψ_1 and Ψ_4 respectively. On the other hand, the current passing through the capacitance C links the total flux. The current circulating in the shorting loop (i.e. the figure 8 loop at the top) has different directions on the two sides. Then the fluxes in different groups are:

$$\begin{cases} \Psi_1 = - L_1 (Is_1 + Ic + I_l) \\ \Psi_2 = - L_2 (Ic + I_l) \\ \Psi_3 = - L_3 (Ic - I_l) \\ \Psi_4 = - L_4 (Is_2 + Ic - I_l) \end{cases} \tag{18}$$

Due to the shorting loop, the fluxes in both sides are forced to be equal, namely:

$$\Psi_1 + \Psi_2 = \Psi_3 + \Psi_4 \tag{19}$$

then

$$(Is_1L_1 - Is_2L_4) + Ic(L_1+L_2-L_3-L_4) + I_l(L_1+L_2+L_3+L_4) = 0 \tag{20}$$

In the symmetric case $L_1 = L_4$ and $L_2 = L_3$, then

$$L_1 (Is_1 - Is_2) = -2 I_l (L_1 + L_2) \tag{21}$$

In push-pull mode, $Is_1 = Is_2$. (Note we've already defined the different direction in the figure.) Then $I_l = 0$. This means if the structure is perfectly symmetric, there will be no current in the figure 8 loop. Otherwise, I_l will begin flowing and balance any existing asymmetry.

The voltages at the output of the amplifier and at the capacitance are:

$$Vs_1 = -\frac{\partial \Psi_1}{\partial t} = j\omega L_1 (Is_1 + Ic) \tag{22}$$

$$Vc = -2\frac{\partial}{\partial t}(\Psi_1 + \Psi_2) = 2 \cdot j\omega[L_1 Is_1 + (L_1 + L_2) Ic] \tag{23}$$

Considering the symmetry, the above equations are equivalent to the circuit shown in Fig.14(a). The right figure (b) is a half, with $C_2 = 2C$. A more delicate equivalent circuit is shown in Fig.14(c).

$$\begin{cases} r_2 = \omega L_2 / Q \\ R_l = Q\omega L_l \end{cases} \tag{24}$$

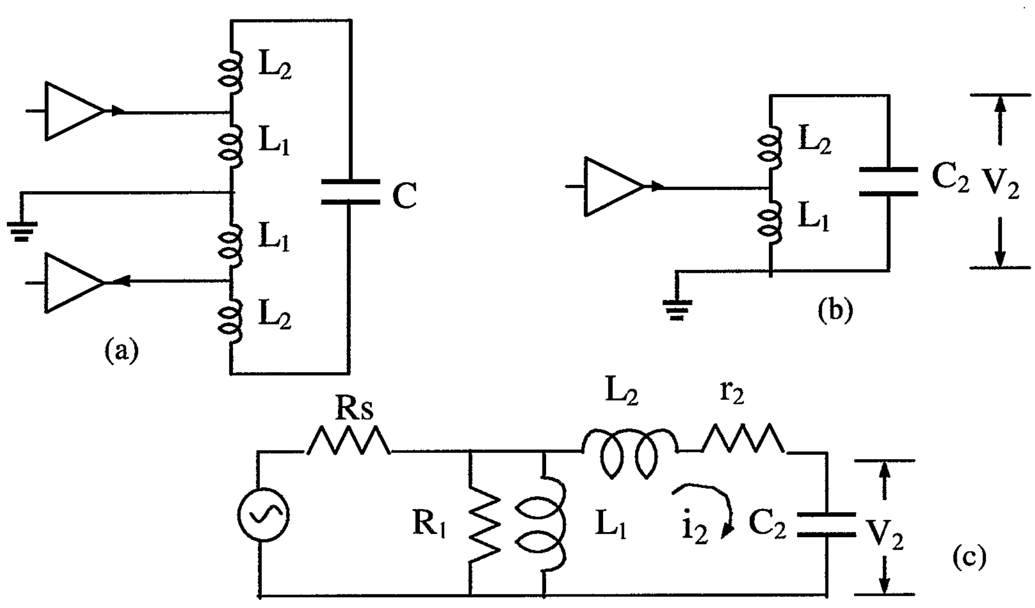


Fig.14 Equivalent circuit of inductance coupling

Comparing with Fig.10, one can see that the two capacitors are replaced by two inductors while the inductance is replaced by a capacitance, so their performances are similar. In particular, the voltage enhancement factor is not proportional to the ratio of the two inductances. In addition, the bandwidth is rather narrow, so that it is hard for one cavity to cover the full band without significantly increasing the power requirement. Therefore, we are forced to use more cavities with staggering.

Note that Fig.14 is not a self-coupling transformer. A self-coupling transformer should have their fluxes fully linked, but that is hard to realize physically in this cavity structure. The figure 8 loop makes coupling, but only balances the asymmetry of the mechanical structure.

Nevertheless, one can modify the structure that makes use of the "figure 8" loop to link them and form a transformer. Fig.15 shows possible structures with different voltage step ratios.

In Fig.15 (a), the ferrite rings are divided into two equal groups with one "figure 8" loop, that forces them to have equal flux. Thus the gap voltage will be doubled. Similarly, in Fig.15 (b) there are three equal groups of ferrite with two "figure 8" loops to force them to have equal flux and the total voltage will be tripled. Fig.15 (c) has four equal groups and two "figure 8" loops forming a 1:4 transformer with the same principle.

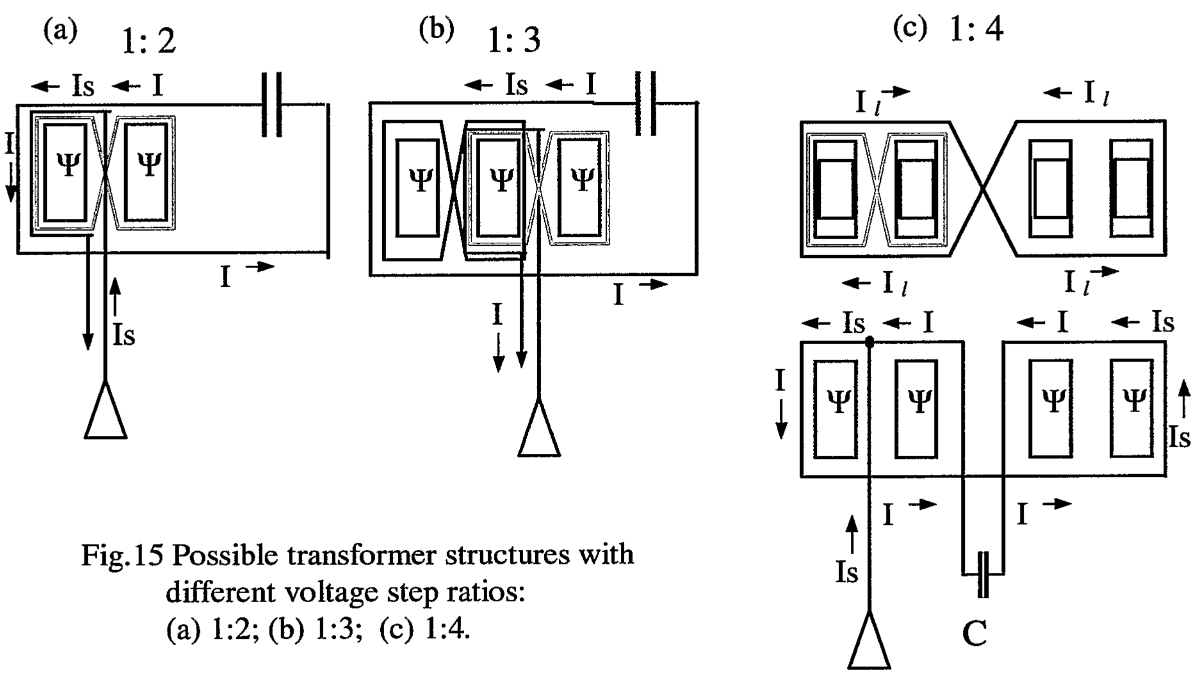


Fig.15 Possible transformer structures with different voltage step ratios: (a) 1:2; (b) 1:3; (c) 1:4.

Of course, the "figure 8" loops make the mechanical structure complicated .

ROTATING CAPACITOR TUNING

A tunable cavity can always be operated on resonance and thus saves large amounts of power. Since the repetition rate is 30 Hz, mechanical tuning is possible. Fig. 16 shows the circuit for capacitance tuning. The reason for employing two tuning capacitors will be explained shortly.

Generally, mechanical tuning is not favorable because of its reliability and operation maintenance. Normally, mechanical tuning structures can not run rapidly and its constituent components are complex, easy to fatigue, have limited lifetime, need replacement regularly, e.g. bellows have limited reciprocating number. In addition, moving electrical contact points will cause corrosion, friction will cause wearing, etc.

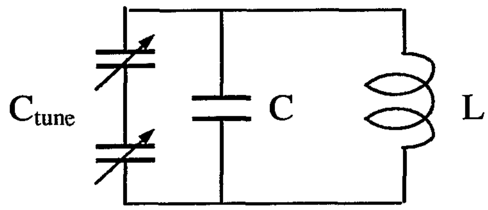


Fig.16 A cavity with tunable capacitors

But, on the other hand, if the structure is so designed that is simple without wearing parts, it is also reliable in modern technology. For example, a motor can work years without failure. Many facilities involving mechanical vacuum pumps can operate unattended. A rotating capacitor can meet the needs and remain reliable.

According to the frequency ramp[1], assuming the inductance $L=15\ \mu H$, the corresponding capacitance tuning is shown in Fig.17. The capacitance change rate, i.e. dC/dt, is shown in Fig.18.

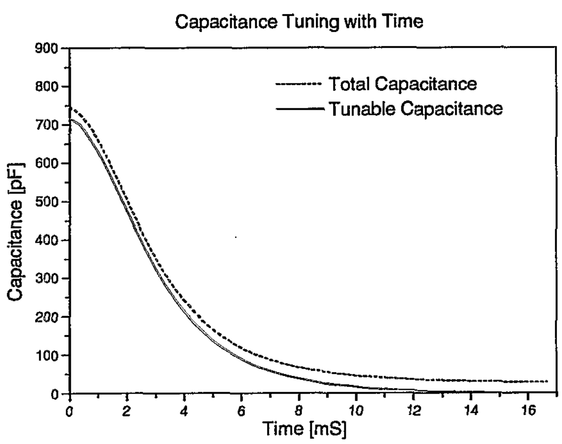


Fig.17 Capacitance tuning curve

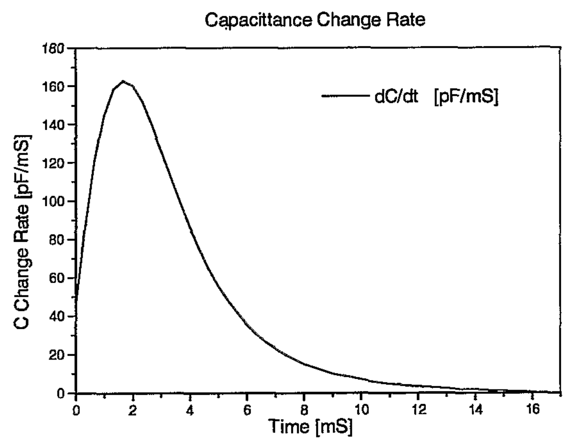


Fig.18 Capacitance change rate

The schematic structure of a tunable capacitor is shown in Fig.19. Two fixed electrodes are located at the top and bottom respectively, while on the center is a rotating plate. There is a gap between them to form a capacitance. As is well known, the capacitance of a parallel plate capacitor is proportional to its area, so the capacitance will change periodically when the plate is rotated.

The plate is shaped in such a way that the area will change in time to fit the requirement shown in Fig.18. There is a stack of electrode pairs to increase the

[1] The calculation in this section is based on an early version where the ramp is from 1.5 MHz to 7.6 MHz.

capacitance to meet the need. Modification is also possible, e.g. one can shape the fixed electrodes instead of the rotating plate if that eases the machining and the capacitance trimming.

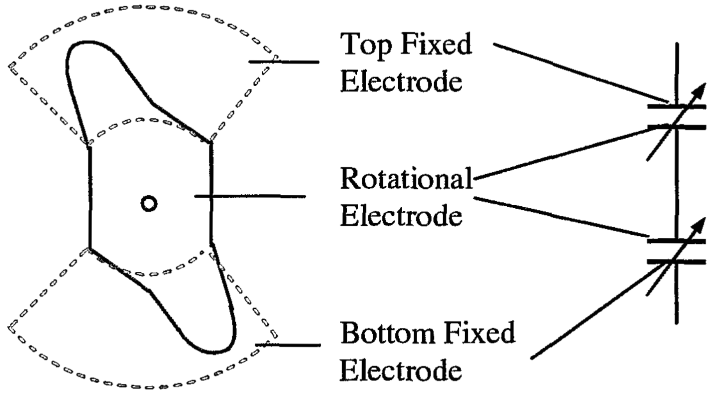


Fig.19 Schematic of a rotating capacitor

As seen in Fig.16 and Fig.19, there are two capacitors in series. This design has a few advantages. The symmetrical shape is good for mechanics. The rotating plate can be electrically floating to avoid the moving electric contact point. The voltage is divided into two halves to alleviate the insulation requirement. One turn of rotation corresponds to two cycles of capacitance tune, so that the rotation rate is reduced to a half, i.e. 15 Hz, or 900 RPM, which is quite modest for a motor. Fig.20 shows the capacitance tune curve for one turn. On the duty time, as in the 1st and 3rd quadrant, the capacitance conforms to the requirement of Fig.17. The shapes in other quadrants don't matter, since they correspond to recovery only.

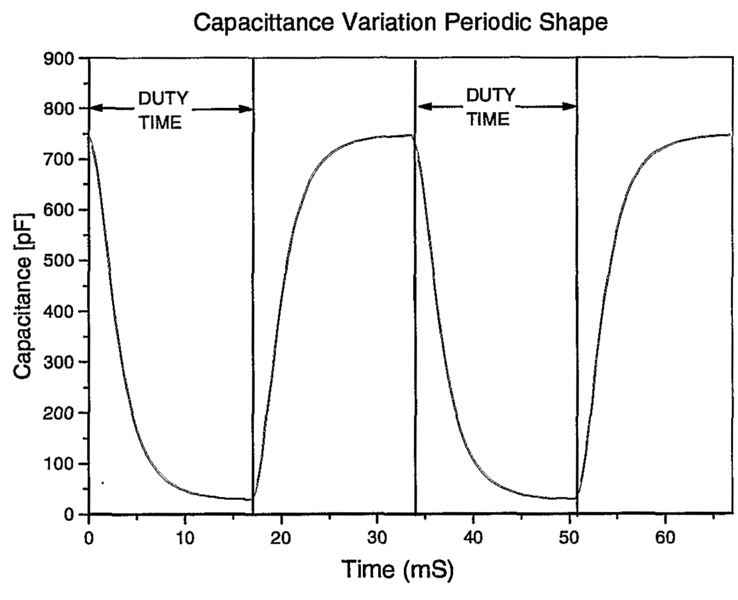
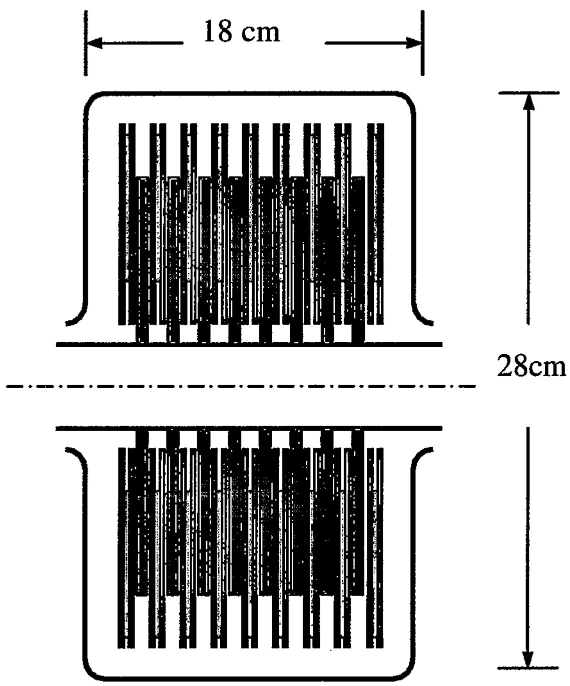


Fig.20 The capacitance tune cycle.

A schematic design shown in Fig.21 gives an idea about the assembly and the size. The fixed electrode is sandwiched between two rexolite thin plates. This improves the insulation and mechanical rigidity. The gap of capacitance has 2mm of air plus 2.5mm of rexolite, which should be adequate to sustain a 3.5 kV voltage. The maximum diameter



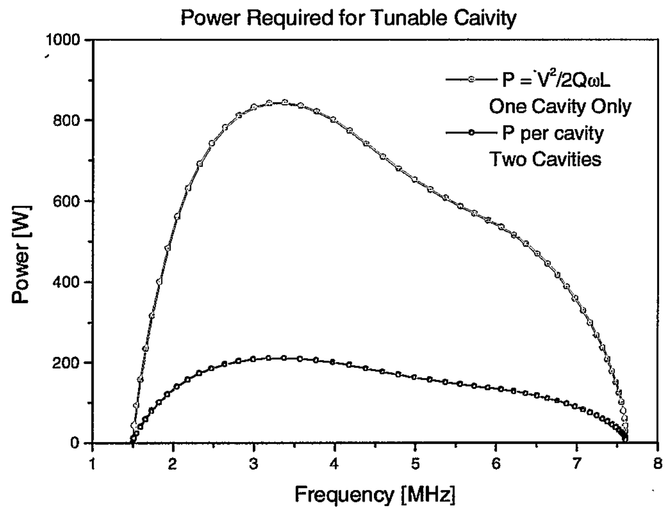


Fig.21 The schematic assembly of the rotational capacitor

Fig.22 The power dissipated inside cavity.

of the rotation plate is only 20 cm. The total assembly extent is about 28 cm. It is compact. The insulation for the rotating parts is not detailed in this figure.

Fig. 22 shows the required power according to the following formula:

$$P = \frac{V^2}{2Z_{sh}} = \frac{V^2}{2Q\omega L} \tag{25}$$

where $L = 15\ \mu H$, $Q = 80$ and V conforms the ramp requirement. If one employs two cavities, the required power for each reduces to a quarter, as is also shown in Fig.22. The total power saving is about 80 times, which is conceivably the same order of Q. Therefore one cavity is enough to do the job and the second cavity can be a backup.

FERRITE BIASING TUNING

Besides mechanical tuning, one can also apply electrical tuning, which seems, in many cases, more preferable. Ferrite is the most popular material for this purpose. Although at the beginning 30 Hz was considered too high for the ferrite to accommodate, we finally found it was still a very worthy option.

A separate note[6] gives a detailed discussion about this option and thus is omitted here.

FILTER OPTION -- IMPEDANCE MATCHING NETWORK

The bandwidth of a simple cavity is narrow. As is well known, a filter can widen it and function as a wide band impedance converter to improve the impedance match. According to network theory, a cavity has only one pole, while a filter consisting of multi elements has multi poles. Therefore, properly arranging the pole distribution can widen the band. Fig.23 shows the schematic.

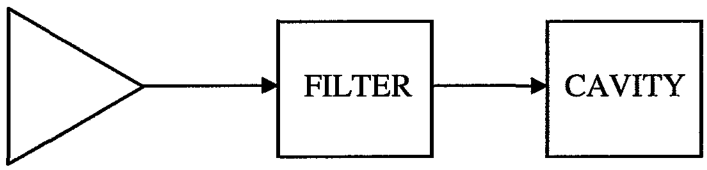


Fig. 23 A filter inserted to improve the bandwidth

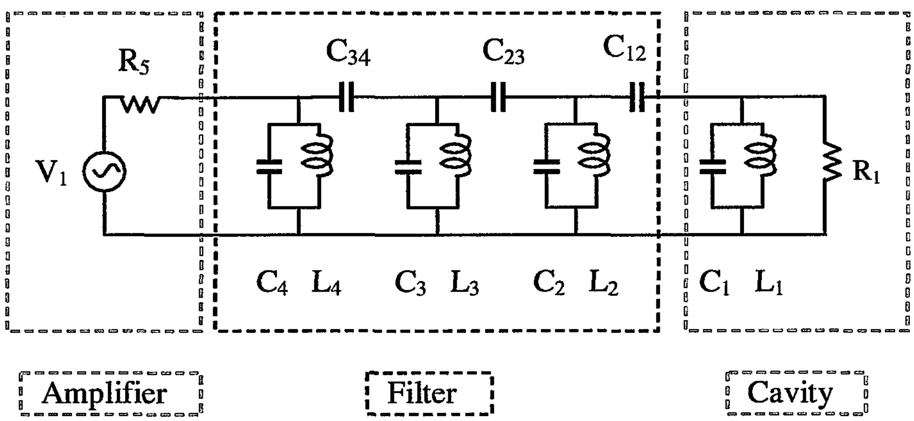


Fig.24 A filter structure

The filter theory has been developing for many decades. However, it is found difficult to cover a bandwidth ratio 1:5 and a loaded Q value about 80 or so, as required in our case. Lowering Q may increase the band, but meanwhile increases the power requirement. To this end, one has to divide it into two or more sub-bands. Many possible options have been studied. Some selected results are summarized below.

Fig.24 shows an impedance-matching network with number of sections $n = 4$. The first section (the right most) is the cavity itself and is a part of the filter. L_1 is the inductance of the ferrite rings, while C_1 is the gap capacitance plus an external capacitance. The load R_1 is its ferrite loss plus possible external resistance for the sake of lowering the Q.

The required frequency ramp is from 1.27 MHz to 6.44 MHz. The center frequency is its geometric mean, namely 2.86 MHz. In the two-band case we need two cavities, one to cover low band and the other high band. The design process is described in the Appendix.

After the parameters were determined, simulations were done using PSpice to check the response. Parameters can then be trimmed for optimization. Some typical results are shown in Fig.25 and Fig.26 with their parameters listed in Table 5.

Table 5

	Case 1	Case 2	Case 3	Case 4	
R_1	20k	9.57 k	20 k	20 k	9.57 k
L_1	25 μ	20 μ	9.0 μ	9.0 μ	19 μ
C_1	200 p	120 p	80 p	75 p	100 p
C_{12}	166 p	187 p	94 p	94 p	187 p
L_2	11.8 μ	12.7 μ	5.3 μ	5.3 μ	12.7 μ
C_2	376 p	180 p	59 p	59 p	180 p
C_{23}	343 p	360 p	200 p	200 p	360 p
L_3	5.54 μ	6.43 μ	2.2 μ	2.2 μ	6.43 μ
C_3	479 p	32 p	120 p	120 p	32 p
C_{34}	1056 p	1030 p	540 p	540 p	1030 p
L_4	2.8 μ	3.3 μ	1.5 μ	1.5 μ	3.3 μ
C_4	2940 p	1760 p	720 p	720 p	1780 p

In Fig.25, the left plots show the amplitude response providing the source voltage V1 = 1 volt. As can be seen the voltage gain is 3 or more. The right plots show the source currents. Because PSpice assumes that a constant voltage source is current unlimited, one has to check if the current is within the limitation of a real source.

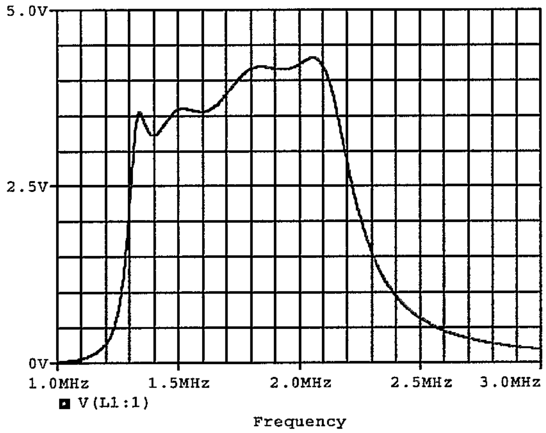
At first we tried 4 sub-bands with the lowest band of 1.27 to 1.91 MHz, but found the resultant bandwidth is wider than expected. It is shown in Fig.25-Case 1. It suggests that two sub-bands would be enough. On the other hand, the required ramp voltage shown in Fig.2 is not flat, implying the flatness is not required. A modified design of a band of 1.27-2.18 MHz is shown in Case 2. Also shown is the voltage V_{L4} at the output of the power source (blue curve).

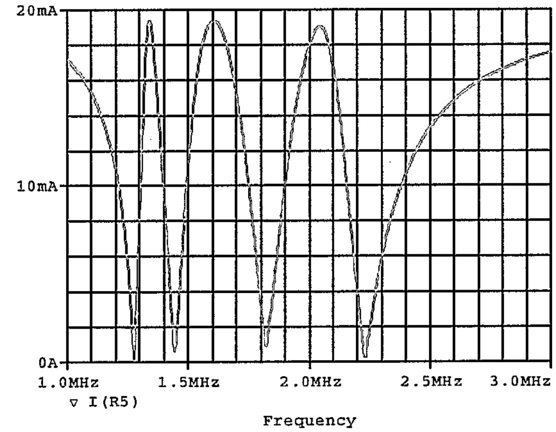
The blue curve V_{L4} has four peaks, corresponding to four poles. Similarly, the current curve reveals four minimums or four zeros (right red curve).

Case 3 is a design for high frequency band.

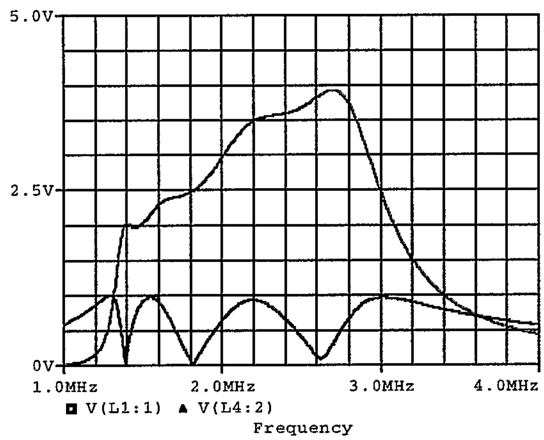
Case 4 shown in Fig.26 is a combination of two cavities with two filters with low band and high band each. It covers the required full band. The peak in the middle is due to overlap. The extra gain showing poor flatness is not important because it can be compensated by adjusting the drive power in the low level system.

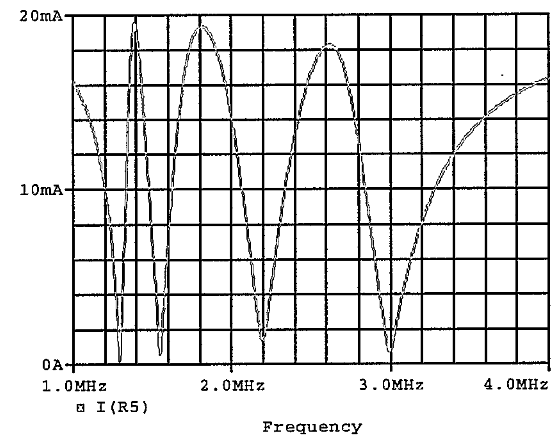
Case 1



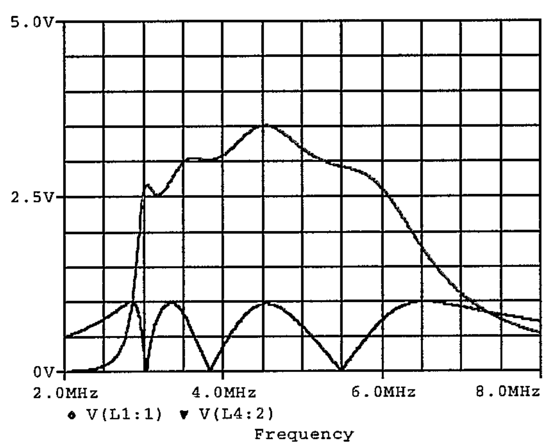


Case 2





Case 3



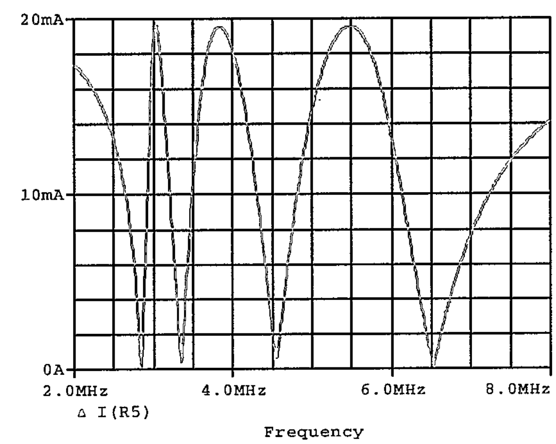


Fig.25 Filter response with different parameters (Left plot --voltage gain, right plots -- source current

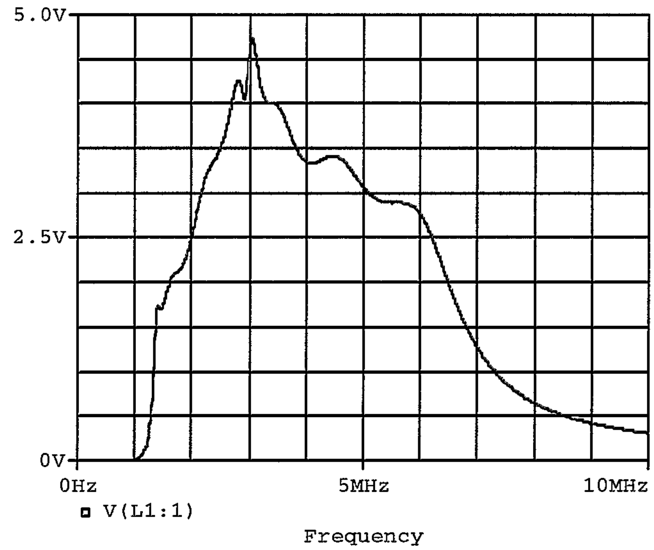


Fig. 26 A combined curve of two cavities
(Case 4 in Table 5)

In conclusion, the filter option is one possible solution without a tuning structure. The power requirement is listed in Table 6 for comparison.

SUMMARY AND COMPARISON OF COST ESTIMATION

Technically all options are realizable. The major restriction in practice would be the cost. The major difference in the cost is its power requirement, or the volume of the amplifier. The following table lists the estimated cost of the amplifier for different options.

Table 6

Option	# of Cav.	# of Gap	Amplifier	Total Cost of amp. (est)	Realizability	Note
Tube	1	1	2×100kW	~ $400k	Certainly	Need maintenance
Capacitor coupling	3	6	6×5kW	$1350k	Yes	
	2	4	4×5kW	$900k	May be	After optimization
Inductance coupling	3	3	6×5kW	$1350k	Yes	
	2	2	4×5kW	$900k	May be	After optimization
Rotational capacitor	1	1	1×2.5kW	$125k	Yes	Need experiment
	2	2	2×1kW	$184k or less	Yes	
Ferrite biasing tune	1	1	1×2.5kW	$125k	Yes	Need experiment
	2	2	2×1kW	$184k or less	Yes	
Filter	3	6	6×5kW	$1350k	Yes	
	2	4	4×5kW	$900k	May be	After optimization

Listed also are the number of cavities and gaps. The two cavities option looks acceptable. The three cavities option is surely not preferable.

It is clear that a tunable scheme will significantly reduce the cost. These include the rotational capacitor option and the ferrite tuning option. The mechanical tuning by a rotational capacitor is, to my point of view, promising, though it needs an experiment to confirm its mechanical realizability. This wouldn't be a serious problem for state-of-art components, though it was a problem decades ago. The ferrite tuning is also promising, and is discussed indetail in a separate note.[6] The tube option is also acceptable from the viewpoint of cost, but its need of maintenance and the complexity of its supporting system makes it unfavorable.

Many options for driving a cavity with ferrite rings have been discussed. Although eventually the proposal of Rapid Cycling Medical Synchrotron may not be approved, I hope this investigation may serve as a reference for some similar machines.

References

[1] S. Peggs, et al."RCMS -- A second generation medical synchrotron", PAC01, June 2001.

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[3] S. Peggs, "Pre-conceptual design of a rapid cycling medical synchrotron", BNL, C-A/AP/6, Oct. 1999

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[5] Phillips Components data sheet

[6] Y. Zhao, "A study of ferrite cavity", BNL-69153, Apr. 2002.

[7] G. L. Matthaei, L. Young, Leo. Jones, E.M.T., “Microwave filters, impedance-matching networks and coupling structure”, Artech House Books: Dedham, MA 1980 (Reprinte of 1964 edition)

APPENDIX

Appendix. The design for a Tchebyscheff impedance-matching network

I. General and Impedance converter

Following common design procedures, one has to design the low pass filter first, and then the pass-band filter. To convert a filter into an impedance matching network one has to insert impedance inverters J12, J23, and J34 between sections, as shown in Fig.A-1 (a). The parameters of the impedance inverters can then be designed, Fig.A-1(b). The parallel capacitance can be merged together, as shown in Fig.A1(c).

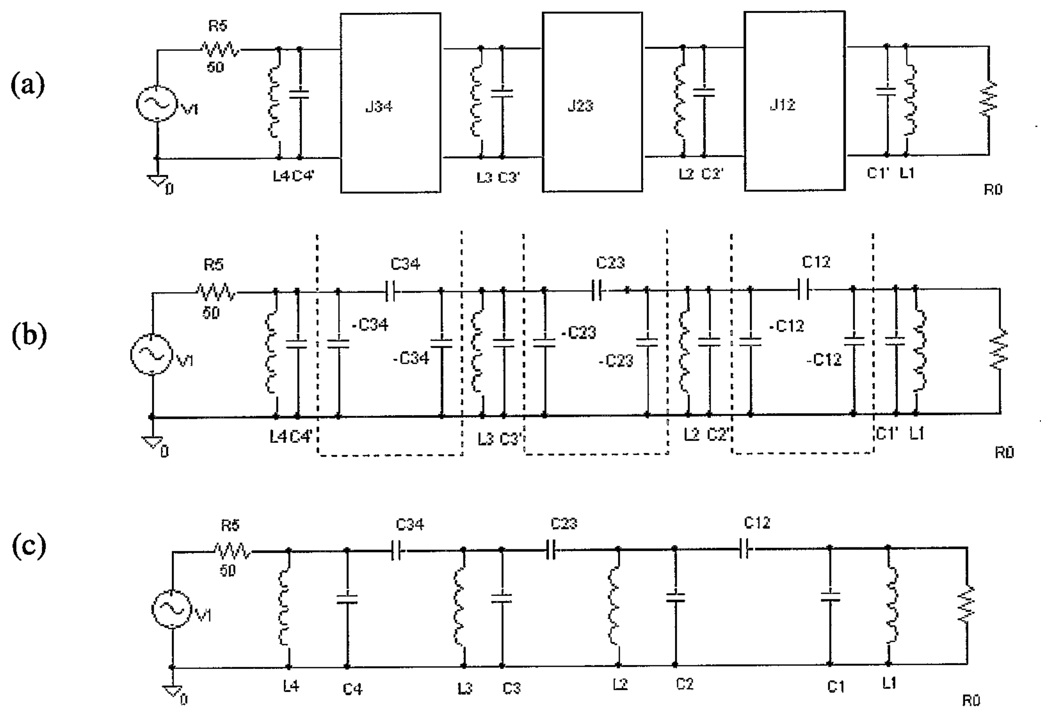

Fig.A-1 The design process of an impedance converter filter

II. Summary of design process

Step 1. Specify number of sections n (here we choose $n = 4$)
Define the frequency span of each sub-band.

Step 2. Specify the basic parameters

Band:	f_{min}, f_{max}
Center frequency:	$f_0 = \sqrt{f_{min} f_{max}}$
Relative bandwidth:	$w = (f_{max} - f_{min}) / f_0$
Q of resonated load:	Q_A
Decrement:	$\delta = 1 / (wQ_A)$

Step 3. Define load and generator parameters

Specify: $R_B = 50\Omega$ -- Source internal resistance, $G_B = 1/ R_B$

L_A -- Load inductance

Calculate: $C_A = 1/\omega_0^2 L_A$, $b_1 = \omega_0 C_A$

$R_A = Q_A \omega_0 L_A$, $G_A = 1/R_A$

Step 4. Calculate low pass prototype parameters g_i (i = 0,1,2,3,4,5)

Normalization: $g_0 = 1$, $g_1 = 1/\delta$, ($\delta = 1/(\omega_1' g_0 g_1$, $\omega_1' = 1$)

To find g_i, there are two ways:

a) Find g_i from existing curves (e.g. reference [7] Fig. 4.09-8)

b) Follow the formulas (see later section III) to calculate

Step 5. Calculate network parameters -- Convert from the low pass parameters

$b_4 = g_4 g_5 G_B / w$

b_1 and b_4 are then defined. b_2,b_3 can be chosen arbitrarily in a certain extent, one may choose with geometric progression that:

$\alpha^3 = b_4 / b_1$, $b_2 = \alpha b_1$, $b_3 = \alpha^2 b_1$

Then: $C_i' = b_i / \omega_0$, $L_i = 1/\omega_0^2 C_i'$

Step 6. Impedance converter parameters

$$J_{12} = \sqrt{\frac{wG_A b_2}{g_1 g_2 \delta}}, \qquad J_{23} = w\sqrt{\frac{b_2 b_3}{g_2 g_3}}, \qquad J_{34} = w\sqrt{\frac{b_3 b_4}{g_3 g_4}}$$

$$C_{12} = J_{12}/\omega_0, \qquad C_{23} = J_{23}/\omega_0, \qquad C_{34} = J_{34}/\omega_0$$

Step 7. Calculate capacitance

$C_1 = C_1' - C_{12}$

$C_2 = C_2' - C_{12} - C_{23}$

$C_3 = C_3' - C_{23} - C_{34}$

$C_4 = C_4' - C_{34}$

Fill all above parameters into the circuit below

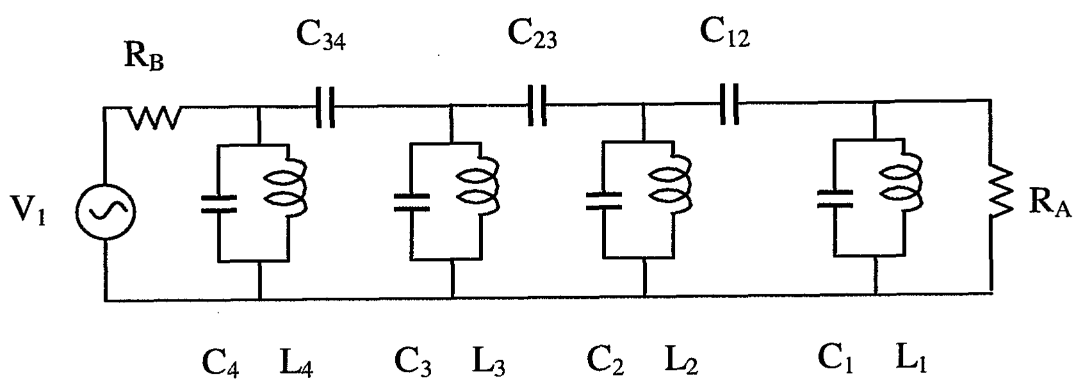

Fig.A-2 The final filter

III. Calculation of the low pass filter parameters g_i

Specify Tchebyscheff ripple value $L'_A = L_{Amax} - L_{Amin}$ (e.g. $L'_A = 1\text{dB}$)

$$H = 10^{(L'_A/10)}$$

$$d = \frac{\sin^{-1}\sqrt{1/(H-1)}}{n}$$

$$D = \frac{d}{\delta \sin\left(\dfrac{\pi}{2n}\right)} - 1$$

$g_0 = 1$, $\qquad g_1 = 1/\delta$,

$$g_j = \frac{1}{g_{j-1}(k_{j-1,j})^2(\omega'_1)^2} \qquad (j = 2 \text{ to } n)$$

$$n = 2, \quad k_{12} = \sqrt{\frac{1+(1+D^2)\delta^2}{2}}$$

$$n = 3, \quad k_{12} = \sqrt{\frac{3}{8}\left[1+\left(1+\frac{D^2}{3}\right)\delta^2\right]}$$

$$k_{23} = \sqrt{\frac{3}{8}\left[1+\left(\frac{1}{3}+D^2\right)\delta^2\right]}$$

$$n = 4, \quad k_{12} = \sqrt{\frac{1}{2\sqrt{2}}\left[1+\left(1+\frac{8D^2}{\alpha^4}\right)\delta^2\right]}$$

$$k_{23} = \sqrt{\frac{2}{\alpha^2}\left[1+\frac{2}{\alpha^2}\left(1+D^2\right)\delta^2\right]}$$

$$k_{34} = \sqrt{\frac{1}{2\sqrt{2}}\left[1+\left(\frac{8}{\alpha^4}+D^2\right)\delta^2\right]}$$

where $\alpha = 2(2+\sqrt{2}) = 6.83$

$$g_{n+1} = \frac{1}{D\delta g_n \omega'_1}$$